Viscous Flowfield Calculations on Pointed Bodies at Angle of Attack in Nonuniform Freestreams

FRANKIE G. MOORE*
U. S. Naval Weapons Laboratory, Dahlgren, Va.
AND
FRED R. DEJARNETTE†
North Carolina State University, Raleigh, N. C.

Theme

THREE-DIMENSIONAL laminar boundary layers are calculated on cones,at incidence in supersonic wake-like freestreams. The nonuniformity of the freestream makes the flowfield nonconical, and therefore both the inviscid and viscous flowfields are three dimensional.

Content

A wake-like nonuniform freestream is represented by the relation

$$V_1/V_\infty = 1 + A[1 - \exp(-By_1^2)] \tag{1}$$

where V_1 is the velocity at the transverse position y_1, V_∞ is the velocity at $y_1 = 0$ (nose of the cone), A is the velocity defect at $y_1 = 0$, and B is a constant which controls the distribution of the nonuniformity (see Fig. 1). Other properties in the freestream are

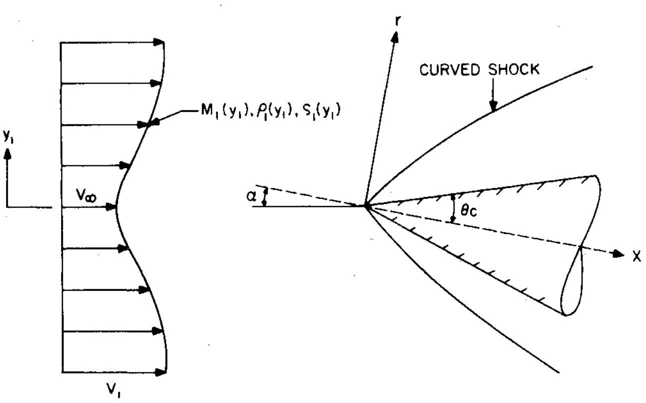

Fig. 1 Coordinate system and freestream properties.

determined from Eq. (1) and the condition of constant static pressure and total enthalpy. The inviscid flowfield is then calculated from a modified form of Rakich's three-dimensional method of characteristics (NASA TN D-5341). Typical results for the non-dimensional pressure ($\bar{p}_b = p_b/\rho_\infty V_\infty^2$) and inviscid surface Mach Number (M_b) are shown in Fig. 2 for a 20° half-angle cone at $\alpha = 10°$ and $M_\infty = 3$. For a velocity defect of $A = 0.2$ the surface Mach number goes to unity at $x = 0.95$ (based on $B = 10$) in the windward plane, and the three-dimensional method of characteristics cannot be continued beyond this point.
Member AIAA.

Submitted June 20, 1971; presented as Paper 71-624 at the AIAA 4th Fluid and Plasma Dynamics Conference, Palo Alto, Calif., June 21–23, 1971; synoptic received September 9, 1971. Full paper is available from AIAA. Price: AIAA members $1.50; nonmembers $2.00. Microfiche $1.00. Order must be accompanied by remittance.

Index category: Boundary Layer and Convective Heat Transfer-Laminar.

* Aerospace Engineer, Surface Warfare Department.

† Professor of Mechanical and Aerospace Engineering. Associate Member AIAA.

Three-dimensional laminar boundary layers are calculated by using the axisymmetric analog or small cross-flow assumption. In order to apply the axisymmetric analog, inviscid surface streamlines and the scale factor, or metric coefficient, h_3 must be calculated first. The scale factor h_3 corresponds to the surface coordinate σ which is orthogonal to the streamlines, and h_3 represents the "equivalent radius." With Φ defined as the circumferential angle on the cone and ϕ the angle between the streamline direction and the generator on the cone, the equation of a streamline becomes

$$D\Phi/Dx = \tan\phi/x \sin\theta_c \tag{2}$$

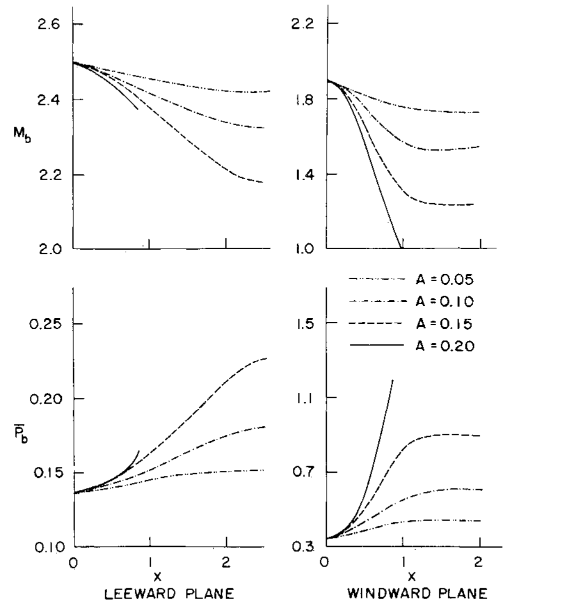

Fig. 2 Surface Mach number and pressure distributions on a cone in axisymmetric wakes; $M_\infty = 3$, $\theta_c = 20°$, $\alpha = 10$.

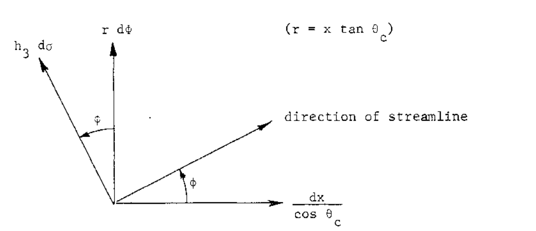

Fig. 3 Streamline coordinates.

where D/Dx is a derivative along a streamline (σ = constant), the differential distance orthogonal to the streamlines is $h_3\, d\sigma$, which is also the spacing between adjacent streamlines.

From Fig. 3 it follows that

$$h_3\, d\sigma = r\, d\Phi \cos\phi - (dx/\cos\theta_c)\sin\phi \tag{3}$$

and with $\Phi = \Phi(\sigma, x)$ Eq. (3) gives

$$(\partial\Phi/\partial\sigma)x = (h_3/r\cos\phi) \tag{4}$$

By interchanging the order of differentiation, the following equation is formed.

$$\frac{D}{Dx}\left(\frac{\partial\Phi}{\partial\sigma}\right)_x = \left[\frac{\partial}{\partial\sigma}\left(\frac{D\Phi}{Dx}\right)\right]_x = \left[\frac{\partial}{\partial\Phi}\left(\frac{D\Phi}{Dx}\right)\right]_x\left(\frac{\partial\Phi}{\partial\sigma}\right)_x \tag{5}$$

Finally, substitute Eqs. (2) and (4) into Eq. (5) to obtain the differential equation for the scale factor as

$$\frac{D}{Dx}\left[\ln\left(\frac{h_3}{r\cos\phi}\right)\right] = \frac{1}{x\sin\theta_c}\left[\frac{\partial}{\partial\Phi}(\tan\phi)\right]_x \tag{6}$$

Eqs. (2) and (6) are integrated numerically to determine the angular position Φ and the scale factor h_3, respectively, along a streamline. The streamline-direction angle ϕ is obtained from the three-dimensional method of characteristics solution on the surface.

Using the axisymmetric analog, laminar boundary-layer properties are calculated along each inviscid surface streamline, independently of the others, by Blottner's numerical method.[1] For the special case of a cone at incidence in a uniform freestream, the present method was found to calculate skin-friction coefficients (C_f) and heating rates (q_w) reasonably close to both experimental data and an "exact" numerical solution.

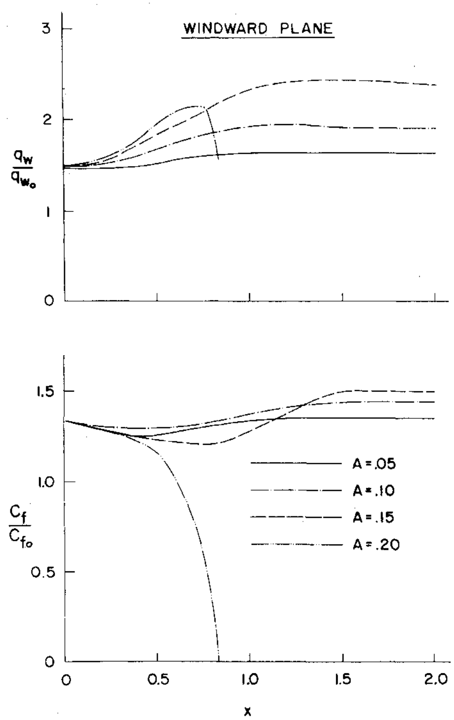

Fig. 4 Normalized local heat transfer and skin-friction coefficient on a cone; $M_\infty = 3$, $\theta_c = 20°$, $\alpha = 10°$, $R_n/\text{ft} = 2.85 \times 10^5$, $T_w/T_o = 0.4$.

Figure 4 shows the distribution of q_w/q_{w_o} and C_f/C_{f_o} along the windward plane for the same conditions as the cone in Fig. 2 (C_{f_o} and q_{w_o} refer to values for this cone at $\alpha = 0$ in a uniform freestream). In Fig. 2 the surface Mach number was found to go to unity at $x = 0.95$ for $A = 0.2$, and Fig. 4 indicates boundary-layer separation at $x = 0.8$ (upstream of the $M_b = 1$ position). However, the separation point is affected by the ratio of wall to stagnation temperature (T_w/T_o), and separation did not occur upstream of the $M_b = 1$ position for cold walls, i.e. $T_w/T_o < 0.2$. On the other hand, separation was always found to occur upstream of the $M_b = 1$ position for cool to hot walls, i.e. $T_w/T_o > 0.4$. The Reynolds number per foot (R_n/ft) did not affect the separation point.

Figure 5 depicts the circumferential variation of q_w/q_{w_o} and C_f/C_{f_o} at $x = 0.75$ (based on $B = 10$) for several angles of attack.

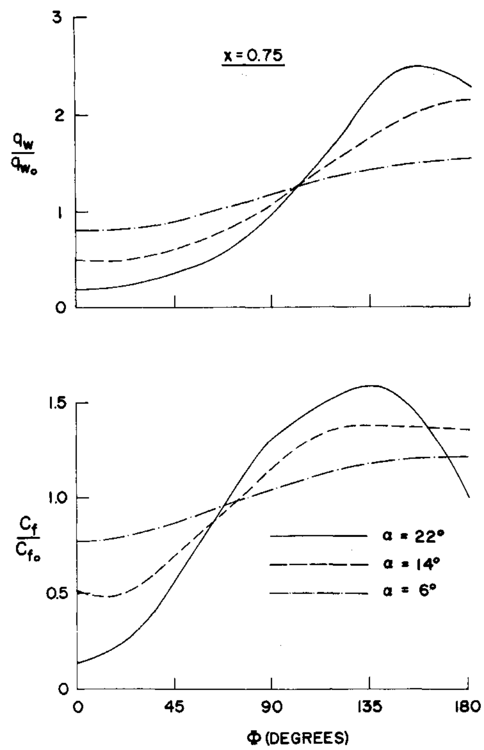

Fig. 5 Normalized local heat transfer and skin-friction coefficient on a cone; $M_\infty = 3$, $\theta_c = 20°$, $\alpha = 14°$, $A = 0.1$, $R_n/\text{ft} = 2.85 \times 10^5$.

The windward plane is $\Phi = 180°$, and the leeward plane is $\Phi = 0$. For $\alpha = 22°$ both q_w and C_f reach a maximum around the cone away from the windward plane.

In conclusion, the axisymmetric analog for three-dimensional laminar boundary-layers was found to yield accurate q_w and C_f on pointed cones at incidence in uniform freestreams. Calculations for cones at incidence in wake-like nonuniform freestreams indicated an adverse pressure gradient on the surface which reduced the inviscid Mach number to unity in some cases. In addition, boundary-layer separation was found to occur in some examples.

Reference

[1] Blottner, F. G., "Finite Difference Methods of Solutions of the Boundary-Layer Equations," *AIAA Journal*, Vol. 8, No. 2, Feb. 1970, pp. 193–205.